# Department of Mathematics SCHOOL OF PHYSICAL SCIENCES



CENTRAL UNIVERSITY OF KERALA केरल केन्द्रीय विश्वविद्यालय

# M.SC. & PH.D (MATHEMATICS) PROGRAMME

CURRICULUM & SYLLABUS

# **Programme Name: MSc Mathematics**

# **Objectives:**

**Technical Proficiency:** Victorious in getting employment in different areas, such as, industries, laboratories, educational, research institutions since the impact of the subject concerned is very wide.

**Professional Growth:** Keep on discovering new avenues in the chosen field and exploring areas that remain conducive for research and development

**Management Skills:** Encourage personality development skills like time management, crisis management, stress interviews and working as a team.

# **Programme Outcomes:**

The general programme out come of M. Sc Mathematics is summarized as follows:

- a. Inculcate critical thinking to carry out scientific investigation objectively without being biased with preconceived notions.
- b. Apply knowledge of Mathematics, in all the fields of learning including research and its extensions
- c. Equip the student with skills to analyze problems, formulate an hypothesis, evaluate and validate results, and draw reasonable conclusions thereof.
- d. Prepare students for pursuing research or careers in industry in mathematical sciences and allied fields

e. Imbibe effective scientific and/or technical communication in both oral and writing. f. Continue to acquire relevant knowledge and skills appropriate to professional activities and demonstrate highest standards of ethical issues in mathematical sciences. g. Create awareness to become an enlightened citizen with commitment to deliver one's responsibilities within the scope of bestowed rights and privileges., and

- h. Inculcate mathematical reasoning and logics and also to develop problems solving capability.
- i. Work effectively as an individual, and also as a member or leader in multi-linguistic and multi-disciplinary teams.

j. Effectively communicate about their field of expertise on their activities, with their peer and society at large, such as, being able to comprehend and write effective reports and design documentation, make effective presentations

# **Programme Specific Outcomes:**

# Listed below are the programme specific outcomes of M. Sc Mathematics

a. Provide advanced knowledge on topics in pure and applied mathematics, empowering the students to pursue higher degrees at reputed academic institutions. b. Prepare and motivate students for research studies in mathematics and related fields. c. Nurture problem solving skills, thinking and creativity through assignments, project works etc.

- d. Assist students in preparing (personal guidance, books) for competitive exams e.g. NET, GATE, etc
- e. Provide knowledge of a wide range of mathematical techniques and application of mathematical methods

# **Programme Name: PhD Mathematics**

# **Programme Outcomes:**

At the end of the programme, the students will be able to:

- a. Identifying unsolved yet relevant problem in a specific field.
- b. Articulating ideas and strategies for addressing a research problem.
- c. Undertaken original research on a particular topic.
- d. Effectively communicating research, through journal publications and conference presentations, to the mathematics community and,
- e. Disseminating research to a broader audience

# **Program Specific Outcomes:**

- a. Generate publications in reputed mathematical journals
- b. Provide scope for interaction with international researchers and developing collaborations
- c. Demonstrate the highest standard of ethics in research

d. Provide opportunities to research students for communication (and discussion) of advanced mathematical topics to undergraduate and graduate students and, e. Produce next generation researchers in mathematics.

# CENTRAL UNIVERSITY OF KERALA DEPARTMENT OF MATHEMATICS

# PROGRAMME STRUCTURE OF TWO YEAR M.SC. (MATHEMATICS)

Course	Title	L	т	P/PD	С
Semester I					
MAT 5101	Real Analysis	4	1	0	4
MAT 5102	Elementary Number Theory and Basic Algebra	4	1	0	4
MAT 5103	Linear Algebra	4	1	0	4
MAT 5104	Discrete Mathematics	4	1	0	4
MAT 5105	Тороlоду	4	1	0	4
Semester-2					
MAT 5201	Algebra	4	1	0	4
MAT 5202	Complex Analysis	4	1	0	4
MAT 5203	Measure and Integration	4	1	0	4
MAT 5204	Multivariable Calculus	4	1	0	4
MAT 5205	Ordinary Differential Equations	4	1	0	4
Semester-3					
MAT 5301	Functional Analysis	4	1	0	4
MAT 5302	Partial Differential Equations	4	1	0	4
MAT 5303	Numerical Analysis	4	1	0	4
MAT 5391	Computational Lab	1	0	2	2
MAT	Elective - I*	4	1	0	4
MAT	Elective - II*	4	1	0	4
Semester-4					
MAT 5490	Dissertation	0	0	18	6

MAT	Elective - III	4	1	0	4
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\*Electives are focused on increasing the self-reading, self-understanding and self-implementing of a research subject and hence contact hours are not mandatory for students who do the course outside the University. Those interested students in III Semester M.Sc. Programme are allowed to opt MOOCs course as their Electives, and shall give the equivalent credit weightage to the students for the credits earned through online learning course through SWAYAM MOOCs platform.

L = Lecture, T = Tutorial, P = Practical, PD=Project Discussion, C = Credits

Credits from Core Courses: 48 – 60

Credits from Elective Courses: 12 - 24

Minimum Total Credits Required: 72

#### LIST OF ELECTIVES

Course	Title	L	т	Р	С
MAT 5001	Algebraic Geometry	3	2	0	4
MAT 5002	Analytic Number Theory	3	2	0	4
MAT 5003	Commutative Algebra	3	2	0	4
MAT 5004	Cryptography	3	2	0	4
MAT 5005	Differential Geometry	3	2	0	4
MAT 5006	Dynamical Systems	3	2	0	4
MAT 5007	Ergodic Theory	3	2	0	4
MAT 5008	Fixed Point Theory	3	2	0	4
MAT 5009	Fluid Dynamics	3	2	0	4
MAT 5010	Fourier Analysis	3	2	0	4
MAT 5011	Galois Theory	3	2	0	4
MAT 5012	Game Theory	3	2	0	4
MAT 5013	Mathematical Finance	3	2	0	4
MAT 5014	Mathematical Methods	3	2	0	4

MAT 5015	Operator Theory	3	2	0	4
MAT 5016	Operations Research	3	2	0	4
MAT 5017	Optimization Techniques and Control Theory	3	2	0	4
MAT 5018	Probability Theory	3	2	0	4
MAT 5019	Queuing Theory	3	2	0	4
MAT 5020	Stochastic Models and Applications	3	2	0	4
MAT 5021	Topological Dynamics	3	2	0	4
MAT 5022	Topological Groups	3	2	0	4
MAT 5023	Introduction to Distribution Theory	3	2	0	4

# LIST OF ELECTIVES OFFERED TO OTHER DEPARTMENTS

Course	Title	L	Т	Р	С
MAT 5051	Probability Theory	3	2	0	4
MAT 5052	Operations Research	3	2	0	4

# VALUE ADDED COURSES

### **Duration of each Course : 30 hours**

Course Title	Operations Research
Course Details	Basic Concept of Linear and Non Linear Programming problem, Application of linear and Nonlinear programming problem, Evaluation of critical Path, project evaluation and review techniques. Game theory and its application.
Faculty	Dr. K. A Germina
Course Title	Introduction to LaTeX and Scientific Writing

Course Details	Environment Setup for LATEX, Basic component of LATEX, formatting and layouts, figures and tables, basic mathematics, Resume & Report Writing, Thesis Writing, Paper Writing, PPT preparation.
Faculty	Dr. Gnanavel S
Course Title	Basics of MATLAB for Scientific Computing
Course Details	Root finding and equation solving, Solving system of equations, Eigenvalues, eigenvectors and eigen decomposition, Singular Value Decomposition, Interpolation, curve fitting and surface modeling, Numerical integration and differentiation, Working with polynomials, Solving Ordinary Differential Equations (ODEs), Solving Boundary Value Problems (BVPs), Solving Delayed Differential Equations (DDEs), Linear Programming (LP), Mixed-Integer Linear Programming (MILP), Quadratic Programming (QP), Constrained and unconstrained nonlinear optimization. Familiarity with parallel open course
Faculty	Dr. Manikandan Rangaswamy /Dr. Gnanavel S.
Course Title	Basic Calculus for Scientists and Economists
Course Details	Numbers, Functions, Sequences and Limits of Functions, Continuity, Derivative, Maxima and Minima and Taylor's expansion, Integration of Real Functions, Function of two variables, Continuity and Differentiability, Lagrange Multiplier Rule, Infinite Series and Multiple Integrals.
Faculty	Dr. Manikandan Rangaswamy
Course Title	Basics in Linear Algebra
Course Details	Euclidean vector spaces, Eigenvalues and eigenvectors,

	Orthogonal matrices, Linear transformations, Solving systems of equations with matrices, Mathematical operations with matrices, Matrix inverses and determinants, Numerical Linear Algebra.
Faculty	Dr. Shaini P
Course Title	BASIC MATHEMATICAL ANALYSIS
Course Details	A quick review of sets and functions and planned to build a basic knowledge In the following: Mathematical induction. Finite and infinite sets. Real Numbers. The algebraic property of real numbers. Absolute value and real line. The completeness property of R. Applications of supremum property Intervals, Nested interval property and uncountability of R. Sequence of real numbers Sequence and their limits Limit theorems Monotone sequences Subsequence and Bolzano – Weirstrass theorem Cauchy criterion Properly divergent sequences. Open and closed sets. Sums and Products. Basic Algebraic properties; Further properties, Vectors and Moduli; Complex conjugates; Exponential form; Product and powers in exponential form; Arguments of products and quotients; Roots of complex numbers; Regions in the complex plane.
Faculty	Dr Ali Akbar K

Corse Code MAT 5101: Real Analysis	L	Т	Ρ	Credit
Prerequisites: Calculus.	4	1	0	4

Course Category	Core
Course Type	Theory

Course Objective	This course presents a rigorous treatment of fundamental concepts in analysis. To introduce students to the fundamentals of mathematical analysis and reading and writing mathematical proofs. The course objective is to understand the axiomatic foundation of the real number system, in particular the notion of completeness and some of its consequences; understand the concepts of limits, continuity, compactness, differentiability, and integrability, rigorously defined; Students should also have attained a basic level of competency in developing their own mathematical arguments and communicating them to others in writing.
Course Outcome(s)	Describe the fundamental properties of the real numbers that underpin the formal development of real analysis; demonstrate an understanding of the theory of sequences and series, continuity, differentiation and integration; Demonstrate skills in constructing rigorous mathematical arguments; Apply the theory in the course to solve a variety of problems at an appropriate level of difficulty.

Real number system and its order completeness. Sequences and series of real numbers. Metric spaces: Basic concepts, continuous functions, Intermediate Value Theorem, Compactness, Heine-Borel Theorem.

Differentiation, Taylor's theorem, Riemann Integral, Improper integrals, Sequences and series of functions, Uniform convergence, power series, Fourier series, Weierstrass approximation theorem, equicontinuity, Arzela-Ascoli theorem.

#### Text books:

- 1. W. Rudin, Principles of Mathematical Analysis, McGraw-Hill, 1976.
- 2. Robert Gardner Bartle and Donald R. Sherbert, Introduction to Real Analysis, 4<sup>th</sup> Edition, Wiley, 2011.

#### **References:**

1. C.C. Pugh, Real Mathematical Analysis, Springer, 2002.

- 2. T. M. Apostol, Mathematical Analysis, 2nd Edition, Narosa, 2002.
- 3. G. F. Simmons, Introduction to Topology and Modern Analysis, McGraw-Hill,
- 1963. 4. Stephen Abbot, Understanding Analysis, Springer, New York, NY, 2015

Code:MAT5102: Elementary Number Theory and Basic	L	Т	Ρ	Credit
Algebra Prerequisites: Number systems.	4	1	0	4

Course Category	Core
Course Type	Theory
Course Objective	Introduce the basic concepts of Number theory such as Divisibility, Congruences, Congruences with Prime Modulus, Quadratic reciprocity and some functions of Number Theory; introduce basic structures of algebra like groups, rings, fields and vector spaces which are the main pillars of modern mathematics.
Course Outcome(s)	Generate facility in working with situations involving commutative rings, in particular monogenic algebras of matrices a concept that finds a large number of applications. Students will see and understand the connection and transition between previously studied mathematics and more advanced mathematics. The students will actively participate in the transition of important concepts such homomorphisms & isomorphisms from discrete mathematics to advanced abstract mathematics. The course gives the student a good mathematical maturity and enables to build mathematical thinking and problem solving skill.

Basic representation theorem, the fundamental theorem of arithmetic; Combinatorial and Computational number theory. Permutations and combinations, Fermat's little theorem, Wilson's theorem. Generating functions; Fundamentals of congruences – Residue systems, Ring; Solving congruences – Linear congruences, Chinese remainder theorem, Polynomial congruences.

Plane Isometries, Direct products & finitely generated Abelian Groups, Binary Linear Codes, Factor Groups, Factor-Group Computations and Simple Groups, Series of groups. Group action on a set, Applications of G-set to counting, Isomorphism theorems: Proof of the Jordan-Holder Theorem, Sylow theorems, Applications of the Sylow theory, Free Groups, Group representations.

# Text books:

1. Thomas Koshy, Elementary Number Theory with Applications, Elsevier, 2007. 2. Joseph Gallian, Contemporary Abstract Algebra, 7th Edition, Cengage Learning, 2009.

#### **References:**

 George E. Andrews: Number Theory, Dover Publications, New York, 1971.
Tom M. Apostol, Introduction to Analytic Number Theory, Springer, 1998.
M. Artin: Algebra, Prentice Hall, 1991.
I. N. Herstein, Topics in Algebra, John Wiley & Sons; 2nd Edition, 1975.
Thomas W. Hungerford, Algebra ,Springer, 2003. 6. John B. Fraleigh, A First Course in Abstract Algebra, 7th Edition, 2002.

Code:MAT5103: Linear Algebra	L	Т	Ρ	Credit
Prerequisites: <b>Basics in Matrix Theory:</b>	4	1	0	4

Course Category	Core
Course Type	Theory
Course Objective	To provide a solid foundation in the mathematics of linear algebra. To develop problem solving skills To prepare the students for advanced level of Mathematics To discuss some of the applications of linear algebra

Course Outcome(s)	The students are: equipped with standard concepts and tools in linear algebra that they would find useful in their disciplines; made competent enough to pursue advanced level of Mathematics; enabled to use Linear Algebra techniques when it is required; get an insight into the enormous applicability of linear algebra. The competency developed include: Solving systems of linear equations; Qualitative analysis of systems of linear equations; Develop understanding of vector Spaces, linear independence , determinants, canonical forms , familiarize analysis of Transformations and use of eigen values and decomposition techniques.
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Vector Spaces: subspaces, bases and dimensions, co-ordinates, summary of row equivalence. Linear Transformations: Linear transformation, the algebra of linear transformation, Isomorphisms, representation of transformations by matrices. Linear Transformations (contd) : Linear functionals, the double dual, the transpose of linear transformations.

Determinants: Commutative Rings, Determinant functions, Permutation and the uniqueness of determinants, Additional properties of determinants. Elementary Canonical Forms: Introduction, characteristic values, annihilating polynomials, invariant subspaces, simultaneous triangulation, simultaneous diagonalisation, direct sum decomposition, invariant direct sums, Jordan, Rational form and diagonalization.

#### Text books:

1. Kenneth Hoffman and Ray Kunze, Linear Algebra, 2nd Edition, Prentice Hall of India Private Ltd, New Delhi, 1971.

# **References:**

 Gilbert Strang, Introduction to Linear Algebra, Wellesley-Cambridge Press; 5<sup>th</sup> Edition, 2016.
Klaus Janich, Linear Algebra, Springer Verlag, 1994.

3. Paul R. Halmos, Linear Algebra Problem Book, The Mathematical Association of America, 1995.

4. Kumaresan, Linear Algebra: A Geometric Approach, Prentice Hall, 2000.

Code:MAT5104: Discrete Mathematics	L	Т	Ρ	Credit
Prerequisites: Set theory and logic: Basic concepts, cardinal numbers	4	1	0	4

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Course Type	Theory
Course Objective	Prepare students to develop mathematical foundations to understand and create mathematical arguments, require in learning many
Course Outcome(s)	It develop ability to deal with notions of mapping and via that notion ability to tackle various notions of infinity like countable, uncountable etc. ; ability to unifying theme for various combinatorial problems, and apply combinatorial intuitions in network theory, data structure and various other fields of science.

Set theoretic operations and functions - Countable and uncountable sets - Mathematical induction - Binary relations - Pigeonhole principle -Discrete numeric functions, Generating functions, recurrence relations.

Lattices as algebraic systems - Principles of duality - Basic properties – Distributive and complemented lattices - Boolean lattices - Boolean algebra - Boolean functions and expressions.

Introduction to Graphs: The concept of a graph, Graphs and graph models, special types of graphs - path, trail, way, cycle, circuit, regular graphs, bipartite graphs, complete graphs, external graphs, intersection graphs. Graph Isomorphism, self-complementary graphs. Representing Graphs: Adjacency matrix, incidence matrix, cycle matrix. Blocks, cut-points, bridges and blocks. Trees - Properties of trees - BFS Algorithm. Eulerian Graphs, Hamiltonian Graphs. Coverings and Matching: Coverings and independence, critical points and lines, matching, maximum matching problems, minimum covering problems. Planar graphs: Plane and planar graphs, outerplanar graphs, Kuratowski's theorem - coloring problems - basic ideas.

# Text books:

1. Norman L. Biggs, Discrete Mathematics, Oxford University Press, 2002.

2. Frank Harary, Graph Theory, Narosa Publishing House, 2001.

# **References:**

- 1. C. L. Liu, Elements of Discrete Mathematics, McGraw-Hill, 2000.
- 2. Douglas B West, Introduction to Graph Theory, Prentice Hall, 2008.
- 3. Paul R. Halmos, Naive Set Theory, Dover Publications Inc.; Reprint Edition, 2017.

Code:MAT5105: Topology	L	Т	Р	Credit
Prerequisites: Basic Knowledge in Set theory and Real Analysis at Undergraduate level	4	1	0	4

Course Category	Core
Course Type	Theory
Course Objective To prepare the students to understand the meaning of a topology to study various other concepts of Topological spaces.	
Course Outcome(s)	Understanding continuity in general settings, Understand Open bases and open sub bases, Weak topologies, the function algebras; Discuss Tychonoff's theorem, locally compact spaces, Compactness of metric spaces and Ascoli's theorem; Distinguish Urysohn's lemma and the Tietze extension theorem; Discuss connected spaces, the components of a space and Totally disconnected spaces; Study Stone-Weierstrass theorems and its applications

Topological Spaces, Basis for a topology, Subspace topology, Closed sets and Limit points, Continuous Functions, Product Topology, Quotient Topology. Connected spaces, Connected subspaces of the Real line, Components and Local Connectedness, Path connectedness, Compact spaces, compactification, Limit-point compactness, Local compactness.

Countability and Separation axioms, Urysohn Lemma, Urysohn Metrization Theorem, Tietze Extension Theorem, Tychonoff Theorem.

# Text books:

1. J.R. Munkres, Topology, 2nd Ed., Pearson Education India, 2001.

# **References:**

1. K.D. Joshi, Introduction to General Topology, New Age International, New Delhi,

2000. 2. J. Dugundji, Topology, Allyn and Bacon Inc. 1966.

3. J.L. Kelley, General Topology, Van Nostrand, 1955.

4. M.G. Murdeswar, General Topology, New Age International, 1990.

5. G. F. Simmons, Introduction to Topology and Modern Analysis, McGraw-Hill, 1963.

Code:MAT5201: Algebra	L	Т	Ρ	Credit
Prerequisites: Algebra.	4	1	0	4

Course Type	Theory
Course Objective	Gain knowledge in fields in the theory of numbers, groups, Rings, UFD, PID ED Modules, Splitting fields and Galois theory.
Course Outcome(s)	Understanding abstract structures such as groups, rings, etc and algebraic constructions; Understand the concepts of direct product of

groups, normal subgroups, and factor groups; Describe the structure of finite Abelian group; Use Sylow's theorems to describe the structure of certain finite groups; Explain the notion of an extension of a field;
Describe the structure of finite fields, Use Galois theory to analyze the solvability of polynomials, Produce rigorous proofs of
propositions/theorems arising in the context of abstract algebra.

Rings - definition, basic concepts and examples. UFDs, PIDs, Euclidean domains, Gausss Lemma. The Eisenstein criterion, examples and applications. Gaussian primes. Algebraic integers.

Integers in quadratic fields. Rings of polynomials, Factorization of polynomials over a field, Non-commutative examples, Homomorphism and factor rings. Prime and Maximal ideals.

Modules: Definitions and Examples, Direct sums, Free Modules, Quotient Module, Homeomorphisms, Module over PIDs. Introduction to Extension Fields, Algebraic Extensions, Geometric Constructions, Finite Fields, Automorphisms of Fields, The Isomorphism Extension Theorem, Splitting Fields, Separable Extensions, Galois Theory, Illustration of Galois Theory, Insolvability of the Quintic.

#### Text books:

1. John B. Fraleigh, A First Course in Abstract Algebra, 7th Edition, 2002.

2. M. Artin: Algebra, Prentice Hall, 1991.

# References:

1. Thomas W. Hungerford, Algebra, Springer, 2003.

2. John B. Fraleigh, A First Course in Abstract Algebra, 7th Edition, 2002.

3. Joseph Gallian, Contemporary Abstract Algebra, 7th Edition, Cengage Learning,

2009. 4. D.M. Burton, A First Course in rings and ideals, Addison-Wesley, 1970.

5. C. Musili, Introduction to Rings and Modules, Narosa Publishing House, 2001.

Code:MAT5202: Complex Analysis		Т	Ρ	Credit
Prerequisites: fundamental Ideas and theorems about Complex plane power series residues	4	1	0	4

Course Category	Core
Course Type	Theory
Course Objective	The objective of this course is to introduce the fundamental ideas of the functions of complex variables and developing a clear understanding of the fundamental concepts of Complex Analysis such as analytic functions, complex integrals and a range of skills which will allow students to work effectively with the concepts.

Conformal mapping, Linear transformations, cross ratio, symmetry, oriented circles, families of circles, use of level curves, elementary mappings and Riemann surfaces.

Complex integration, rectifiable curves, Cauchy's integral theorems for rectangle and disc, Cauchy's integral formula, higher derivatives. Local properties of analytic functions, removable singularities, Taylors theorem, Taylor series and Laurent series, zeroes and poles, local mapping, the maximum principle. Chains and cycles, simple connectivity, locally exact differentials, multiply connected regions, residue theorem, argument principle, evaluation of definite integrals

Harmonic functions, mean value property, Poissons formula, Schwarz theorem, reflection principle, Weierstrass theorem.

#### Text books:

1. L.V. Ahlfors, Complex Analysis, Third Edition Mc-Graw Hill International, 1979. 2. H. A. Priestley, Introduction to Complex Analysis, Oxford University Press, 2003.

# **References:**

1. John M. Howie, Complex Analysis, Springer Science & Business Media, 2003. 2. John B. Conway, Functions of One Complex Variable I, Springer Science & Business Media, 1978.

3. J. Brown and R. Churchill, Complex Variables and Applications, McGraw-Hill Education, 2013.

4. V. Karunakaran, Complex Analysis, CRC Press, 2005.

5. Dennis G. Zill, Patrick Shanahan, Patrick D. Shanahan, A First Course in Complex Analysis with Applications, Jones & Bartlett Learning, 2006.

Code:MAT5203: Measure and Integration		Т	Ρ	Credit
Prerequisites: Basic knowledge of differentiation, integration and continuity of real functions	4	1	0	4

Course Category	Core
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Course Type	Theory
Course Objective	Knowledge gained about Lebesgue theory and general measure spaces and their properties and construction.

Course Outcome(s)	On completion of the module a student should be able to know and understand the concept of a sigma-algebra and a measure; understand
	the concept of the Lebesgue measure and almost everywhere prevailing properties; Begin with Understanding integration in a general setting using measures; Understand the Radon-Nikodym theorem and relation
	between convergence of Lebesgue integrals and pointwise convergence of functions, products measures and Fubini's theorem.

Review of Riemann Integral, Lebesgue Measure; Lebesgue Outer Measure; Lebesgue Measurable Sets. Measure on an Arbitrary Sigma- Algebra; Measurable Functions; Integral of a Simple Measurable Function; Integral of Positive Measurable Functions.

Lebesgue's Monotone Convergence Theorem; Integrability; Dominated Convergence Theorem; Lp- Spaces. Signed Measures and the Hahn -Jordan Decomposition- Radon-Nikodym theorem and its applications. Differentiation and Fundamental theorem for Lebesgue integration Product measure; Fubini's theorem

#### Text books:

1. G. de Barra, Measure and Integration, 2nd Edition, New Age International publications, 2013. 2. H.L. Royden, Real Analysis, 3rd Edition, Prentice-Hall of India, 1995.

# **References:**

1. W. Rudin, Real and Complex Analysis, Third edition, McGraw-Hill, International Editions, 1987.

2. Inder K. Rana, An Introduction to Measure and Integration, American Mathematical Society, 2005.

3. P. R. Halmos, Measure Theory, Van Nostrand, 1950.

4. D.L. Cohn, Measure Theory, Birkhauser, 1997.

5. P.K. Jain and V.P. Gupta, Lebesgue Measure and Integration, New Age International, 2006.

Code:MAT5204: Multivariable Calculus		Т	Ρ	Credit
Prerequisites: Linear Algebra, Single variable Calculus		1	0	4

Course Category	Core
Course Type	Theory

Course Objective	The objective is to enable the students to develop a clear understanding of the fundamental concepts of multivariable calculus and a range of skills such as the ability to compute derivatives using the chain rule, ability to set up and solve optimization problems involving several variables, with or without constraints, ability to set up and compute multiple integrals in rectangular, polar, cylindrical and spherical coordinates, allowing them to work effectively
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	with the concepts.). This course also envisages to enable the students to understand the major theorems: the Green's, Stokes' and the Gauss' theorems of the course and some physical applications of these theorems.
Course Outcome(s)	Understand the basic concepts and know the basic techniques of differential and integral calculus of functions of several variables; Apply the theory to calculate the gradients, directional derivatives, arc length of curves, area of surfaces, and volume of solids; Solve problems involving maxima and minima, line integral and surface integral and understand the major theorems: the Green's, Stokes' and the Gauss' theorems of the course and some physical applications of these theorems. Develop mathematical maturity to undertake higher level studies in mathematics and related fields.

Functions of several variables, Directional derivative, Partial derivative, Total derivative, Jacobian, Chain rule and Mean value theorems, Interchange of order of differentiation, Higher derivatives, Taylor's theorem, Inverse mapping theorem, Implicit function theorem, Extremum problems, Extremum problems with constraints, Lagrange's multiplier method.

Multiple integrals, Properties of integrals, Existence of integrals, iterated integrals, change of variables.

Curl, gradient, divergence, Laplacian. Cylindrical and spherical coordinates. Line integrals, surface integrals, Theorems of Green, Gauss and Stokes.

# Text books:

1. C.H. Edwards Jr., Advanced Calculus of Several Variables, Academic Press, 1973. 2. Apostol T.M., Calculus-II - Part-2, Non-Linear Analysis

#### **References:**

- 1. Apostol T.M., Mathematical Analysis, Original Edition .
- 2. Apostol T.M., Calculus-II Part-2, Non-Linear Analysis.

Code:MAT5205: Ordinary Differential Equations	L	Т	Ρ	Credit
Prerequisites: Knowledge of ordinary differential equations of first order and second order	4	1	0	4

Course Category	Core
Course Type	Theory
Course Objective	Introduce the concepts of existence and uniqueness of solution of differential equations Develop analytical techniques to solve differential equations Understand the properties of solution of differential equations
Course Outcome(s)	Understand the genesis of ordinary differential equations. Classify the differential equations with respect to their order and linearity; explain the

uniqueness theor linear differentia Analyze real-wo equations (ODEs) about the scenar problems using m	ution of a differential equation; express the existence rem of differential equations; find solution of higher-order l equations; solve systems of linear differential equations. orld scenarios to recognize when ordinary differential ) or systems of ODEs are appropriate, formulate problems rios, creatively model these scenarios in order to solve the nultiple approaches, judge if the results are reasonable, and d clearly communicate the results.
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Ordinary Differential Equations: Linear Equations with constant coefficients – Second order Homogeneous equations - Initial value problems - Linear dependence and independence, Wronskian and a formula for Wronskian - Non-Homogeneous equation of order two.

Homogeneous and Non-Homogeneous Equations of order 'n' - Initial value problems annihilator Method to solve a non-homogeneous equation. Algebra of constant coefficients operators.

Linear Equations with variable coefficients - Initial value problems - Existence and Uniqueness Theorems - Solutions to a non-homogeneous equation -Wronskian and Linear dependence reduction of the order of a homogeneous equation - Homogeneous equation with analytic coefficients - the Legendre equation. Linear Equation with regular singular points – Euler Equation - Second order equations with regular singular points - Exceptional cases - Bessel equation. Existence and Uniqueness of solutions to first order equations - Equation with variables separated - Exact Equations - Method of successive approximations - the Lipschitz condition - convergence of the successive approximations and the existence theorem.

First order systems in two variables and linearization: The general phase plane – some population models - Linear approximation at equilibrium points - Linear systems in matrix form. Examples of nonlinear systems, Stability analysis, Liapunov stability, phase portrait of 2D systems, Poincare Bendixon theory, Leinard's theorem.

#### Text books:

1. Coddington, E. and Levinson, N., Theory of Ordinary Differential Equations. McGraw-Hill, New York, 1955.

#### **References:**

1. Eral. A. Coddington, An Introduction to Ordinary Differential Equations, PHL Learning Pvt Ltd, 2009.

2. Lawrence Perko, Differential equations and dynamical systems, Springer, 3rd Edition, 2001. 3. G.F. Simmons: Differential Equations with Applications and Historical notes. Tata McGraw Hill, 2nd Edition, 2003.

4. A. K. Nandakumaran, P. S. Datti and Raju K. George, Ordinary Differential

Equations: Principles and Applications (Cambridge IISc Series), IISc Press, 2017.

5. Hartman, Ordinary Differential Equations, Birkhaeuser, 1982.

Code:MAT5301: Functional Analysis	L	Т	Ρ	Credit

Prerequisites: Linear algebra, Real analysis, Basic topology

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Course Category	Core
Course Type	Theory
Course Objective	To introduce to students the ideas and some of the fundamental theorems of functional analysis.; to show students the use of abstract algebraic/topological structures in studying spaces of functions; to give students a working knowledge of the basic properties of Banach spaces, Hilbert spaces and bounded linear operators; To enable students to understand the idea of duals, adjoints and spectrum of a bounded linear operator.
Course Outcome(s)	Upon completing the course, students will be able to: Recognize the fundamental properties of normed spaces and of the transformations between them; understand and apply fundamental theorems from the theory of normed and Banach spaces, including the Hahn-Banach theorem, the open mapping theorem, the closed graph theorem, and the Stone-Weierstrass theorem; able to Compute the dual spaces of certain Banach spaces; appreciate the role of Zorn's lemma; understand the notions of inner product and Hilbert space; understand the fundamentals of spectral theory; appreciate how functional analysis uses and unifies ideas from vector spaces, the theory of metrics, and complex analysis.

Normed linear space; Banach spaces and basic properties; Heine-Borel theorem, Riesz lemma and best approximation property; Inner product space and projection theorem; Orthonormal bases; Bessel inequality and Parseval's formula; Riesz-Fischer theorem.

Bounded operators and basic properties; Space of bounded operators and dual space; Riesz representation theorem; Adjoint of operators on a Hilbert space; Self adjoint, Normal and Unitary Operators; Examples of unbounded operators; Convergence of sequence of operators.

Hahn-Banach Extension theorem; Uniform boundedness principle; Closed graph theorem and open mapping theorem. Some applications. Invertibility of operators; Spectrum of an operator. Spectral theory of self adjoint compact operators.

#### Text books:

1. B.V. Limaye, Functional Analysis, Second Edition, New Age International, 1996. 2. G. Bachman and L. Narici, Functional Analysis, Academic Press, 1966.

#### **References:**

1. M. Thamban Nair, Functional Analysis: A First Course, Prentice-Hall of India, 2004. 2. B. Bollabas, Linear Analysis, Cambridge University Press, Indian Edition, 1999. 3. Martin Schechter, Principles of Functional Analysis, 2nd Edition, American Mathematical Society, 2001

4. G. F. Simmons, Introduction to Topology and Modern Analysis, McGraw-Hill, 1963. 5. E. Kreyzig, Introduction to Functional Analysis with Applications, Wiley India Private Limited, 2007.

6. A. E. Taylor and D.C. Lay, Introduction to Functional Analysis, 2nd Edition, Wiley, New York, 1980.

Code:MAT5302: Partial Differential Equations		т	Ρ	Credit
Prerequisites: Basic knowledge Calculus, linear algebra, complex analysis, ordinary differential equations	4	1	0	4

Course Category	Со
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Core

Course Type	Theory
Course Objective	Introduce the concepts of existence and uniqueness of solution of differential equations. Develop analytical techniques to solve differential equations Understand the properties of solution of differential equations. Explore decomposition of continuous functions with Fourier Series. Appreciate the complexities and varied techniques for PDEs
Course Outcome(s)	Use knowledge of partial differential equations (PDEs), modelling, the general structure of solutions, and analytic and numerical methods for solutions. Formulate physical problems as PDEs using conservation laws. understand analogies between mathematical descriptions of different (wave) phenomena in physics and engineering. Classify PDEs, apply analytical methods, and physically interpret the solutions. Demonstrate accurate and efficient use of Fourier analysis techniques and their applications in the theory of PDE's. Apply problem-solving using concepts and techniques from PDE's and Fourier analysis applied to diverse situations in physics, engineering, financial mathematics and in other mathematical contexts.

Partial Differential Equations - First Order Partial Differential Equations - Linear equations of first order. Nonlinear Partial Differential Equations of the first order - Cauchy's method of characteristics - Compatible systems of first order equations - Charpit's method - Special types of First order equations - Jacobis method. Partial Differential Equations of Second order - The origin of Second order Equations, Canonical forms - Linear Partial Differential Equations with constant coefficients - Equations with variable coefficients - Characteristics curves of second order equations - Characteristics of equations in three variables.

The Solution of Linear Hyperbolic Equations - Separation of variables - The Method of Integral

Transforms - Nonlinear Equations of the second order. Elliptic Equation - Occurrence of Laplace Equations in Physics - Elementary solution of Laplace equations - Families of equipotential surfaces, Boundary value problems - Separation of variables - Problems with axial symmetry. Properties of Harmonic functions, Spherical mean - Maximum-minimum principles.

The wave equation - Occurrence of wave equation in Physics - Elementary solutions of one dimensional wave equation - D'Alembert solution - Vibrating Membranes: Applications of the calculus of variations, Duhamel's principle - Three dimensional problems. The Diffusion Equations: Elementary solutions of the Diffusion Equation - Separation of variables - Maximum minimum principles - The use of Integral transforms.

#### Text books:

1. N. Sneddon, Elements of Partial Differential Equations, Dover, 2006.

2. Tyn Myint-U and Lokenath Debnath, Linear Partial Differential Equations for Scientists and Engineers, Birkhauser, Boston, 2007.

#### **References:**

1. Fritz John, Partial Differential Equations, Springer, 1991.

2. Walter A. Strauss, Partial Differential Equations: An Introduction, John Wiley & Sons Inc., 2008.

3. Sandro Salsa, Partial Differential Equations in Action: From Modelling to Theory, Springer, 2nd Edition. 2015.

4. Gerald B. Folland, Introduction to Partial Differential Equations. Second Edition, Princeton University Press, 2nd Edition, 1995.

5. Garabedian P. R., Partial Differential Equations, John Wiley and Sons, 1964. 6. Prasad P and Ravindran R., Partial Differential Equations, Wiley Eastern, 1985. 7. Renardy M. and Rogers R. C., An Introduction to Partial Differential Equations, Springer- Verlag, 1992.

Code:MAT5303: Numerical Analysis Prerequisites: Basic knowledge Calculus, linear algebra,	L	Т	Р	Credi t
complex analysis, ordinary differential equations	4	1	0	4

Course Category	Core
Course Type	Theory

Course Objective	Introduce the concepts of existence and uniqueness of solution of differential equations Develop analytical techniques to solve differential equations Understand the properties of solution of differential equations
Course Outcome(s)	Use knowledge of partial differential equations (PDEs), modelling, the general structure of solutions, and analytic and numerical methods for solutions. Formulate physical problems as PDEs using conservation laws. understand analogies between mathematical descriptions of different

(wave) phenomena in physics and engineering. Classify PDEs, apply
analytical methods, and physically interpret the solutions.

Solution of Equations, Linear Systems and Algebraic Eigenvalue Problems. Solution of algebraic and transcendental equations: Fixed-point iteration method, Newton's method; Linear system (Direct methods): Gaussian elimination - Pivoting - LU Decomposition; Vector and Matrix norms - Error Analysis and Condition numbers; Linear system (Iterative methods): Gauss-Jacobi and Gauss-Seidel -Convergence considerations; Eigenvalue problem: Power method - Jacobi for a real symmetric matrix.

Interpolation, Differentiation and Integration. Interpolation: Lagrange's interpolation - Errors in Lagrange's interpolation - Newton's divided differences - Newton's finite difference interpolation - Optimal points for interpolation - Piecewise Interpolation: Piecewise linear and piecewise Cubic Spline interpolation. Numerical differentiation: Numerical differentiation based on interpolation, finite differences, method of undetermined coefficients; Numerical integration: Newton Cotes formula - Gaussian quadrature - Errors in Simpson's rule and Gaussian quadrature - method of undetermined coefficients - quadrature rules for Multiple integrals. Ordinary Differential Equations. Single-step methods - Euler's method and Modified Euler's method, Taylor series method, Runge-Kutta method of fourth order. Multi-step methods: Adams-Bashforth -Adams-Moulton methods - Stability considerations. Two point BVPs: Finite Difference method Linear problems with Dirichlet and derivative boundary conditions – Stiff equations – Eigenvalue problems.

#### Text books:

1. Faires J. D. and Burden R., Numerical Methods, Brooks/Cole Publishing Co., 1998. 2. Jain M. K., Iyenger S. R. K. and Jain R.K., Numerical Methods for Scientific and Engineering Computation, 3rd Edition, New Age, 1993.

#### **References:**

1. Atkinson K. E., An Introduction to Numerical Analysis, Wiley, 1989.

2 Phillips G. M and Taylor P.J., Theory and Applications of Numerical Analysis, 2nd Edition, Elsevier, New Delhi, 2006.

3. Isaacson E. and Keller H. B., Analysis of Numerical Methods, Dover, 1994. 4. Conte S. D. and Carl de Boor, Elementary Numerical Analysis, 3rd Edition, McGraw-Hill Book Company, 1983.

5. Kincaid D. and Chenney W., Numerical Analysis: Mathematics of Scientific Computing, Brooks/Cole Pub. 2nd Edition, 2002.

6. A. Quarteroni, F. Saleri and P. Gervasio, Scientific Computing with MATLAB and Octave, Springer Science & Business Media, 2010.

Sastry S.S, Introductory Methods of Numerical Analysis, Prentice Hall India, 5<sup>th</sup> Edition,
2012. 8. Iserlas A., First course in the numerical analysis of differential equations, Cambridge, 1996.

Code:MAT5391: Computational Lab	L	Т	Ρ	Credit
Prerequisites: Basics in Computer programming	1	0	2	2

Course Category	Core
Course Type	Theory and Practical
Course Objective	Reinforce a structured, top-down approach to formulate and solve problems. 2. Introduce common approaches, structures, and conventions for creating and evaluating computer programs, primarily in a procedural paradigm, but with a brief introduction to object-oriented concepts and terminology. 3. Apply a variety of common numeric techniques to solve and visualize engineering-related computational problems. 4. Introduce the MATLAB software environment.
Course Outcome(s)	Provide students with the background and skills required to numerically simulate and solve problems of approximations and optimizations. This will be a hands-on class with theory accompanied by practical implementation in MATLAB. After a review of programming in MATLAB and basic numerical methods (linear equations, interpolation, numerical differentiation, integration), methods to solve various ordinary and partial differential equations will be covered.

Introduction to basic operators, Functions and Predefined Variables, Defining Variables. Matrices, Matrix Operations. Plotting Graphs - Two-Dimensional Plots - Three-Dimensional Plot., General Commands, Polynomials, Curve Fitting and Interpolation -programming exercise (Numerical Methods) including development of algorithms to solve ordinary differential equations and partial differential equations. Using which Programming you will be taught for this.

The program coding executed using C or C++ programming languages are preferred. However, codes may also use software programs including Matlab/Octave, Mathematica.

LATEX - Introduction, Document preparation - Basic.

#### **References:**

- 1. J. Stoer and R. Bulirsch, Introduction to Numerical Analysis, Springer-Verlag, ISBN 0-387-90420-4.
- 2. A. Quarteroni, F. Saleri and P. Gervasio, Scientific Computing with MATLAB and Octave, Springer, Science & Business Media, 2010.

Prerequisites: Reasonably good understanding about M.Sc. first year courses; especially those related to the project topic

L	Т	*PD	Credit
0	0	18	6
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Course Category	Core
Course Type	Theory
Course Objective	To provide training in scientific skills ; To prepare students for professional training programme or entry level jobs in any area of Mathematics

Course Outcome(s)	Specific learning outcomes for a Master's Dissertation are for the student to demonstrate: Considerably more in-depth knowledge of the major subject/field of study, including deeper insight into current research and development work; Deeper knowledge of methods in the major subject/field of study; A capability to contribute to research and development work; The capability to use a holistic view to critically, independently and creatively identify, formulate and deal with complex issues; The capability to plan and use adequate methods to conduct qualified tasks in given frameworks and to evaluate this work; The capability to create, analyse and critically evaluate different technical/architectural solutions; The capability to critically and systematically integrate knowledge; A consciousness of the ethical aspects of research and development work. Developing capability for undertaking deep study of a specific topic, procuring relevant literature,
	analysing available results, preparation of scientific report etc

No Syllabus can be prescribed for Project work. It will depend on the specific project chosen by the student in consultation with the faculty guide.

Code:MAT5001: Algebraic Geometry	L	Т	Ρ	Credit
Prerequisites: Algebra I & Algebra II, Topology	3	2	0	4

Course Category	Elective
Course Type	Theory
Course Objective	Algebraic geometry is the study of geometric spaces defined by polynomial equations. It is a central topic in mathematics with strong ties to differential and symplectic geometry, topology, number theory, and representation theory. It is also a very important source of examples throughout mathematics. The aim of this course will be to learn algebraic geometry through the study of key examples
Course Outcome(s)	The student: masters fundamental techniques within classical algebraic geometry; is able to argue mathematically correct and present proofs and reasoning; has solid experience and training in reasoning with geometric

#### structures

#### Syllabus:

Varieties: Affine and projective varieties, coordinate rings, morphisms and rational maps, local ring of a point, function fields, dimension of a variety. Curves: Singular points and tangent lines, multiplicities and local rings, intersection multiplicities, Bezout's theorem for plane curves, Max Noether's theorem and some of its applications, group law on a nonsingular cubic, rational parametrization, branches and valuations.

#### Text books:

1. S. S. Abhyankar, Algebraic Geometry for Scientists and Engineers, American Mathematical Society, 1990.

2. I. R. Shafarevich, Basic Algebraic Geometry 1: Varieties in Projective Space, Springer, 2013.

#### **References:**

1. W. Fulton, Algebraic Curves, Benjamin-Cummings Publishing, 1974.

2. J. Harris, Algebraic Geometry: A First Course, Springer-Verlag, 1992.

3. M. Reid, Undergraduate Algebraic Geometry, Cambridge University Press, Cambridge,

1990. 4. R.J. Walker, Algebraic Curves, Springer- Verlag, Berlin, 1950.

Code:MAT5002: Analytic Number Theory	L	Т	Ρ	Credit
Prerequisites: Number theory.	3	2	0	4

Course Category	Elective
Course Type	Theory
Course Objective	The aim of this course will be mastering the students to handle multiplicative functions, to deal with Dirichlet series as functions of a complex variable
Course Outcome(s)	The course will teach students to handle multiplicative functions, to deal with Dirichlet series as functions of a complex variable, and to prove the Prime Number Theorem and simple variants.

Arithmetic functions - Combinatorial study of Phi(n), Formulae for d(n) and sigma(n), Multivariate arithmetic functions, Mobius inversion formula; Primitive roots - Properties of reduced residue systems, Primitive roots modulo p; Prime numbers - Elementary properties of Phi(x), Tchebychev's theorem.

Quadratic Congruences: Quadratic Residues - Euler's criterion, Legenedre symbol, Quadratic reciprocity law; Distribution of Quadratic Residues - Consecutive residues and nonresidues, Consecutive triples of quadratic residues.

Additivity: Sums of Squares - Sums of two squares, Sums of four squares; Elementary Partition Theory - Graphical representation, Euler's partition theorem, Searching for partition identities;

Partition Generating Functions - Infinite products as generating functions, Identities between infinite series and products.

Partition identities - Euler's pentagonal number theorem, Rogers-Ramanujan identities, Series and products identities, Schur's theorem; Geometric Number Theory: Lattice points - Gausss circle problem, Dirichelet's divisor problem.

#### Text books:

1. Tom M. Apostol, Introduction to Analytic Number Theory, Springer, 1998.

#### **References:**

- 1. Weils A., Basic Number Theory, Springer, 1973.
- 2. TIFR Mathematical Pamphlet: Algebraic Number Theory, 1966.
- 3. Artin M., Algebra, Phi Learning Pvt. Ltd, 2011.
- 4. George E.Andrews: Number Theory, Dover Publications, New York, 1971.

Code:MAT5003: Commutative Algebra	L	Т	Ρ	Credit
Prerequisites: Algebra I & Algebra II.	3	2	0	4

Course Category	Elective
Course Type	Theory

Course Objective	The aim of the course is to serve as a first level foundational course in commutative algebra. In this course, the students will be introduced to the algebra of rings and modules. The students will also be exposed to certain modules that possess some special properties and relationships between them
Course Outcome(s)	Gain familiarity with the polynomial ring and be able to perform basic operations with both elements and ideals; Use computational tools, especially Gröbner bases and the Buchberger algorithm, to solve problems in polynomial rings.

Rings and Modules, Localisation of Rings and Modules, Noetherian Rings and Modules, Primary Decomposition, Artinian rings, Integral Extensions, Going-up, Lying-over and Going-down Theorems, Hilbert's Nullstellensatz, Noether's Normalization, Dimension Theorem, Krull's Principal Ideal Theorem, Dedekind Domains.

# Text books:

1. M. F. Atiyah and I. G. Macdonald: Introduction to Commutative Algebra, Sarat Book House, 2007.

### **References:**

1. N. S. Gopalakrishnan: Commutative Algebra, Oxonian Press, 1984.

2. Gregor Kemper, A Course in Commutative Algebra, Springer, 2013.

Code:MAT5004: Cryptography	L	Т	Ρ	Credit
Prerequisites: Number theory.	3	2	0	4

Course Category	Elective
Course Type	Theory
Course Objective	This course is aimed to serve as a first level course to introduce modern cryptography. The students will be exposed to basics of encryption and authentication in the context of symmetric-key and asymmetric-key cryptography

Course	Describe network security services and mechanisms. Symmetrical and
Outcome(s)	Asymmetrical cryptography. Data integrity, Authentication, Digital
	Signatures. Various network security applications, IPSec, Firewall, IDS,
	Web security, Email security, and Malicious software etc.

Divisibility and Euclidean algorithm, congruence, applications to

factoring. Finite fields, Legendre symbol and quadratic reciprocity,

Jacobi symbol.

Cryptosystems, diagraph transformations and enciphering matrices, RSA Cryptosystem.

Primality and Factoring, Pseudo primes, Carmichael number, Primality tests, Strong Pseudo primes, Monte Carlo method, Fermat factorization, Factor base, Implication for RSA, Continued fraction method.

Elliptic curves - basic facts, Elliptic curves over R;C;Q, finite fields. Hasse's theorem (statement), Weil's conjectures (statement), Elliptic curve cryptosystems, Elliptic curve factorization - Lenstra's method.

# Text books:

1. Neal Koblitz, A Course in Number Theory and Cryptography, Graduate Texts in Mathematics, Springer, 1987.

2. Jeffrey Ho\_stein, Jill Pipher and J.H. Silverman, An Introduction to Mathematical Cryptography, Springer, 1st Edition, 2010.

#### **References:**

1. Rosen M. and Ireland K., A Classical Introduction to Number Theory, Graduate Texts in Mathematics, Springer, 1982.

2. David Bressoud, Factorization and Primality Testing, Undergraduate Texts in Mathematics, Springer, 1989.

Code:MAT5005: Differential Geometry	L	Т	Ρ	Credit
Prerequisites Multivariable calculus, Basics of linear algebra, Topology	3	2	0	4

Course Category	Elective
Course Type	Theory
Course Objective	The aim of the course is to provide knowledge of the geometry of curves and surfaces. The course introduces the fundamentals of differential geometry primarily by focusing on the theory of curves and surfaces in three space. The theory of curves studies global properties of curves. The theory of surfaces introduces the fundamental quadratic forms of a surface, intrinsic and extrinsic geometry of surfaces, and the Gauss- Bonnet theorem.
Course Outcome(s)	After completing this course, students should be able to: Understand the basis of notions of the local theory of space curves, and the local theory of surfaces; understand the fundamental theorem for plane curves and of space curves; recognize whether a given curve (resp. surface) is regular or not; compute the curvature and torsion of a regular curve; understand the idea of orientable /non-orientable surfaces; understand the normal curvature of a surface, its connection with the first and second fundamental form and Euler's theorem ; able to find all geodesic curves of the surface; evaluate the principal curvatures, the mean curvature and Gauss curvature of a given surface; able to find the fundamental forms of surfaces.

Curves in Euclidean space: Curves in R3, Tangent vectors, Differential derivations, Principal normal and binomial vectors, Curvature and torsion, Formulae of Frenet.

Surfaces in R<sup>3</sup>: Surfaces, Charts, Smooth functions, Tangent space, Vector fields, Differential forms, Regular Surfaces, The second fundamental form, Geodesies, Weingarten map, Curvatures of surfaces, Orientation of surfaces.

Differentiable manifolds, differentiable maps and tangent spaces, regular values and Sards theorem, vector fields, submersions and immersions.

# Text books:

1. Gray A., Modern Differential Geometry of Curves and Surfaces, CRC Press, 1993. 2. Victor Guillemin and Alan Pollack, Differential Topology, Orient Blackswan, 2017.

#### **References:**

1. Christian Bar, Elementary Differential Geometry, Cambridge University Press, 2010. 2. Sebastin Montiel and Antonio Ros, Curves and Surfaces, American Mathematical Society,

# 2009.

3. do Carmo M. P., Differential Geometry of curves and surfaces, Prentice-Hall, 1976. 4. O'Neill B., Elementary Differential Geometry, Academic press, 1996.

5. Kumaresan S., A course in differential geometry and Lie groups, Texts and Readings

in Mathematics, Hindustan Book Agency, New Delhi, 2002.

6. Andrew H. Wallace, Differential Topology: First Steps, Dover, 2006.

# Code:MAT5006: Dynamical Systems

Prerequisites: Real Analysis, Ordinary Differential Equations

L	Т	Ρ	Credit
3	2	0	4

Course Category	Elective
Course Type	Theory
Course Objective	To introduce the concept of linear and nonlinear dynamical systems 2. To learn the basic ideas and methods associated with dynamical systems, like, evolution of system, fixed points, periodic points, attractors, bifurcation process and stability of the systems 3. To understand the nonlinearity in nature and study of the nonlinear models in engineering and its dynamics
Course Outcome(s)	Learn the general theory of linear ordinary differential equations, including matrix exponential solutions for constant coefficient equations; Learn the basic local existence and uniqueness theory for ordinary differential equations; Learn basic ideas in differential dynamical systems, including stability of orbits, omega limit sets, Lyapunov functions, and invariant sets; Understand the statements of the stable and center manifold theorems; Learn some basic ideas in chaotic dynamics and bifurcation of vector fields

Review of Linear Systems. Dynamical Systems and Vector Field, Fundamental Theorem, Existence and Uniqueness; Continuous dependence of Solutions with initial conditions; extending solutions; global solutions; flow of a differential equation. Stability of Equilibrium Nonlinear sinks, stability, Liapunov functions, Gradient systems; the Poincare Benedixton theorem and applications. Introduction to Discrete Dynamical Systems.

### Text books:

1. Hirsch M. W. and Smale S., DYNAMICAL SYSTEMS, Academic Press, 1974.

### **References:**

1. Holmgren R. A., A first course in discrete dynamics, Springer Verlag, 1994

Code:MAT5007: Ergodic Theory		Т	Ρ	Credit
Prerequisites: Measure theory	3	2	0	4

Course Category	Elective
Course Type	Theory
Course Objective	To study the long term behaviour of dynamical systems (or iterations of maps) using methods developed in Measure Theory, Linear Analysis and Probability Theory.
Course Outcome(s)	At the end of the module the student is expected to be familiar with the ergodic theorem and its application to the analysis of the dynamical behaviour of a variety of examples.

Poinacre's Recurrence Theorem, Hopf's Maximal Ergodic Theorem, Birkoff's Individual ergodic Theorem, von Neumann's Mean Ergodic Theorem. Ergodicity, Mixing, Eigenvalues. Discrete Spectrum Theorem. Ergodic automorphisms of Compact Groups. Conjugacy. Entropy.

## Text books:

1. Peter Walters, An Introduction to Ergodic Theory, Springer, 2005.

## **References:**

- 1. Halmos P. R., Intordoctory Lectures in Ergodic Theory,
- 2. Nadakarni M. G., Ergodic Theory, Hindustan Book Agency, 3rd Edition, 2013.

Code:MAT5008: Fixed Point Theory Prerequisites: Topology and functional analysis	L	Т	Ρ	Credit
	3	2	0	4

Course Category	Elective
Course Type	Theory
Course Objective	The objective of the course is to motivate and equip the students with the basics in topological as well as metric fixed point theory. It also intends to expose the students to some of the interesting applications in fixed point theory and make them understand how this important tool is used in the study of nonlinear phenomena.
Course Outcome(s)	Upon completion of this course : Students will be familiar with some of the classical results in Metric fixed point theory such as Banach Contraction Principal and several other contraction theorems such as Kannan's fixed point theorem, Chatterjea's fixed point theorem etc.; able to understand the concept of measure of noncompactness; able to understand Brower fixed point theorem and its generalizations such as Schauder fixed point theorem and its applications

students will be able to recognize various iteration schemes for
approximating fixed points

The Background of Metrical Fixed Point Theory, Fixed Point Formulation of Typical Functional Equations, Fixed Point Iteration Procedures, The Principle of Contraction mapping in complete metric spaces, Some generalizations of Contraction mapping, A converse of Contraction Principle, some applications of Contraction Principle.

Convexity, Smoothness, and Duality Mappings, Geometric Coefficients of Banach Spaces, Existence Theorems in Metric Spaces, Existence Theorems in Banach Spaces, Approximation of Fixed Points, Strong Convergence Theorems.

Compactness in metric spaces. Measure of noncompactness, Measure of noncompactness in Banach spaces, Classes of special operators on Banach spaces. The Fixed point property, Brower's Fixed point theorem, equivalent formulations, some examples and applications, The computation of fixed points, Schauder's fixed point theorem and its generalizations,

Applications of Fixed Point Theorems.

## Text books:

1. V. Berinde, Iterative approximation of fixed points, Springer-Verlag, Berlin, Heidelberg, 2007.

2. R. P. Agarwal, Maria Meehan and D.O' Regan, Fixed point theory and applications, Cambridge University Press, 2001.

# **References:**

1. V. I. Istratescu, Fixed Point Theory - An Introduction, D. Reidel Publishing Company, Dordrecht, Holland, 1981.

2. K. Goebel and W. A. Kirk, Topics in Metric \_xed point theory, Cambridge University Press, 1990.

3. A. Granas and J. Dugundji, Fixed point theory, Springer Monographs in Mathe matics, 2003. 4. M. A. Khamsi and W. A. Kirk, An Introduction to Metric Spaces and Fixed Point Theory, A Wiley- Interscience Publication, 2001.

5. W. A .Kirk and B. Sims, Handbook of Metric Fixed Point Theory, Kluwer Academic Publishers, 2001.

6. Sankatha Singh, Bruce Watson and Pramila Srivastava, Fixed point theory and best approximation:

The KKM-Map principle, Kluwer Academic Publishers, 1997.

7. E.Zeidler, Nonlinear Functional Analysis and its Applications I: Fixed Point Theorems, Springer-Verlag, New York, 1986.

Code:MAT5009: Fluid Dynamics	L	Т	Ρ	Credit
Prerequisites:	3	2	0	4

Course Category	Elective
Course Type	Theory
Course Objective	To understand the dynamics of real fluids. To acquire the knowledge of solving problems using partial differential
Course Outcome(s)	develop an appreciation for the properties of Newtonian fluids, study analytical solutions to variety of simplified problems, understand the dynamics of fluid flows and the governing non-dimensional

INVISCID THEORY: Introductory Notions; velocity, streamlines and paths of particles, stream tubes a filaments, fluid body; density; pressure; Bernoulli's theorem; differentiation with respect to time; equation of continuity; boundary conditions – kinematica and physical; rate of change of linear momentum, equation of motion of an inviscid fluid.

Euler's momentum theorem, conservative forces, Lagrangian form of the equation of motion, steady motion; energy equation; rate of change of circulation; vortex motion, permanence of vorticity.

TWO DIMENTIONSAL MOTION: Two dimensional functions - stream function, velocity potential, complex potential, indirect approach, inverse function; basic singularities -source, doublet, vortex, mixed flow; method of images - circle theorem, flow past circular cylinder with circulation; aerofoil - Blasius's theorem, lift force.

VISCOUS THEORY: Equations of motion - Stress tensor, Navier-Stokes equations, vorticity and circulation, some exact solutions of Navier-Stokes equations flow between parallel at plates - Couette flow, Plane Poiseuille flow; steady flow in pipes - Hagen- Poiseuille flow.

BOUNDARY LAYER THEORY: Boundary layer concept; boundary layer equations in two dimensional flow; boundary layer along a at plate - Blasius solution, shearing stress, boundary layer thickness, displacement thickness, momentum thickness; Momentum integral theorem for the boundary layer - Von Karman Integral relation, Von Karman Integral relation by momentum law.

### Text books:

1. Batchelor, An Introduction to Fluid Dynamics, Foundation Books, 2005. 2. Frank Chorlton, Textbook of Fluid Dynamics, CBS, 2004.

#### **References:**

1. L. M. Milne-Thomson, Theoretical Hydrodynamics, Dover, 2011.

2. N. Curle and H. J. Davies, Modern Fluid Dynamics Vol - I, Van Nostrand Company

Ltd., London, 1968.

3. S. W. Yuan, Foundations of Fluid Mechanics, Prentice-Hall, 1970.

<b>Code:MAT5010: Fourier Analysis</b> Prerequisites: Familiarity with measure theory and Hilbert`s space		Т	Ρ	Credit
······, -······························	3	2	0	4

Course Category	Elective
Course Type	Theory

Course Objective	To develop the ability of using important tools and theorems to solve concrete problems, as well as getting a sense of doing formal calculations to predict/verify results
Course Outcome(s)	The student will have to know the theoretical concepts introduced in the lectures, construct and discuss examples related to each of them (in such a way to better understand the abstract concepts), write/reconstruct the proofs seen in the lectures or easy variants of those and solve problems on the topics of the course.

Cesaro summability and Abel summability of Fourier series, Mean square convergence of Fourier series, A continuous function with divergent Fourier series, Applications of Fourier series Fourier transform on the real line and basic properties, Solution of heat equation, Fourier transform for functions in Lp, Fourier transform of a tempered distribution, Poisson summation formula, uncertainty principle, Paley-Wiener theorem, Tauberian theorems, Spherical harmonics and symmetry properties of Fourier transform, Multiple Fourier series and Fourier transform on R<sup>n</sup>.

## Text books:

1. E. M. Stein and Rami Shakarchi, Fourier Analysis, An introduction, Princeton University press, 2003.

2. W. Rudin, Functional Analysis, Tata Mcgraw-Hill, 1985.

### **References:**

 H. Dym and H. P. McKean: Fourier Series and Integrals, Academic Press, 1972.
T. W. Krner: Fourier Analysis, Cambridge University Press, 1988.
J. S. Walker: Fourier Analysis, Oxford University Press, 1988.

Code:MAT5011: Galois Theory	L	Т	Ρ	Credit
Prerequisites: Knowledge in Rings, examples of rings, ideals	3	2	0	4

Course Category	Elective
Course Type	Theory

Course Objective	To know Solvability by radicals, Classical ruler and compass constructions.
Course Outcome(s)	Ability to handle Galois groups, abstractly and in explicit examples, by using

a variety of techniques including the Fundamental Theorem of Galois
Theory and presentations of fields; Capacity to explain and work with the
consequences of Galois Theory in general questions of mathematics
addressed in the course, such as insolubility of certain classes of equations
or impossibility of certain geometric constructions; Understand the
statements and proofs of important theorems and be able to explain the
key steps in proofs, sometimes with variation.

Quick review: (Field theory and Compass constructions: Algebraic, Complex algebraic numbers, Number fields; transcendental, separable, normal purely inseparable extensions; finite fields; the Frobenius of a field of positive characteristic; Perfect fields; theorem of the primitive element; Ruler and Compass constructions; constructing regular polygons;)

Galois theory and applications: Group of automorphisms of fields; fundamental theorem of finite Galois Theory; cyclic extensions; solvability by radicals; Kummer theory; Determining the Galois group of a polynomial

Transcendental extensions: Transcendence basis theorem; Luroth's theorem; transcendence of e.

Algebraically closed fields: Existence and uniqueness of an algebraic closure.

### Text books:

1. Ian Stewart, Galois Theory, Chapman and Hall, 2003.

2. Lang S., Algebra, Springer, 2005.

### **References:**

1. Garling D.J.H., A Course in Galois Theory, Cambridge University Press,

1987. 2. Dummit D. S. and Foote R. M., Abstract Algebra, McGraw-Hill, 1986.

3. Jacobson N., Basic Algebra I & II, Dover Publication, 2009.

4. Jacobson N., Lectures on Abstract Algebra Vol III, Springer, 2013.

<b>Code:MAT5012: Game Theory</b> Prerequisites: Probability theory, linear algebra, linear programming, and		Т	Ρ	Credit
calculus.	3	2	0	4

Course Category	Elective					
Course Type	Theory					
Course Objective	This course will meet the following objectives Provide a foundation in the basic concepts of Game Theory Understand Nash's equilibrium Understand Cooperative v/s Non-Cooperative games					
Course Outcome(s)	Gain a proper understanding of game theoretic concepts and modeling:					

Introduction: rationality, intelligence, common knowledge, von Neumann-Morgenstern utilities;

Noncooperative Game Theory: strategic form games, dominant strategy equilibria, pure strategy Nash equilibrium, mixed strategy Nash equilibrium, existence of Nash equilibrium, computation of Nash equilibrium, matrix games, minimax theorem, extensive form games, subgame perfect equilibrium, games with incomplete information, Bayesian games.

Mechanism Design: Social choice functions and properties, incentive compatibility, revelation theorem, Gibbard-Satterthwaite Theorem, Arrow's impossibility theorem, Vickrey-Clarke Groves mechanisms, dAGVA mechanisms, Revenue equivalence theorem, optimal auctions.

Cooperative Game Theory: Correlated equilibrium, two person bargaining problem, coalitional games, the core, the shapley value, other solution concepts in cooperative game theory.

### Text books:

1. Y. Narahari, Game Theory and Mechanism Design, IISc Press and the World Scientific, 2014.

#### **References:**

1. Roger B. Myerson, Game Theory: Analysis of Conict, Harvard University Press, September 1997.

2. Martin J. Osborne, An Introduction to Game Theory, Oxford University Press, 2003.

<b>Code:MAT5013: Mathematical Finance</b> Prerequisites: Probability theory and Differential Equations.	L	Т	Ρ	Credit
	3	2	0	4

Course Category	Elective	
Course Type	Theory	
Course Objective	The primary goal of this course is to teach students some necessary mathematical techniques and how to apply them to the fundamental concepts and problems in financial mathematics and their solution.	
Course Outcome(s)	e main contents include: Introduction to probability theory, random riable, probability density, mean, and variance of a random variable. The plications include interest rate, coupon bonds, arbitrage, Brownian	

motion, geometric Brownian motion for mathematical models on stock price, etc.

## Syllabus:

Introduction to investment securities and financial derivatives, Random walk, Brownian Motion, Geometric Brownian Motion, Interest rates and Present Value Analysis, Pricing Contracts via Arbitrage, Arbitrage Theorem, Black-Scholes Formula, Valuing by expected utility, Exotic Options, Models for Crude Oil data, Autoregressive Models and Mean reversion.

## Text books:

1. S. M. Ross, An Elementary Introduction to Mathematical Finance, 3rd Edition, Cambridge University Press, 2011.

## **References:**

1. John Hull, Options, Futures, and Other Derivatives, 8th Edition, Prentice Hall, 2011. 2. M. Baxter and A. Rennie, Financial Calculus: An Introduction to Derivative Pricing, Cambridge University Press, 1996.

3. Darrell Duffie, Dynamic Asset Pricing Theory, 3rd Edition, Princeton University Press, 2001. 4. Paul Wilmott, Sam Howison and Jeff Dewynne, The Mathematics of Financial Derivatives: A Student Introduction, Cambridge University Press, 1995.

5. J. P. Fouque, G. Papanicolaou and K. R. Sircar, Derivatives in Financial Markets with Stochastic Volatility, Cambridge University Press, 2000.

Code:MAT5014: Mathematical Methods		Т	Ρ	Credit
Prerequisites:	3	2	0	4

Course Category	Elective
Course Type	Theory
Course Objective	The main aim is to make students familiar with Laplace, Fourier transformations, extrema of functional through calculus of variations and integral equations.

Course Outcome(s)	The course students will be able to recognize difference between Volterra and Fredholm Integral Equations, First kind and Second kind, homogeneous and inhomogeneous etc. They apply different methods to solve Integral Equations. Students will have much better and deeper understanding of the fundamental concepts of the space of admissible variations and concepts of a weak and a strong relative minimum of an
	integral.

INTEGRAL TRANSFORMS: Laplace transform: Definition - properties - Laplace transforms of some elementary functions - Convolution Theorem - Inverse Laplace transformation - Applications. Fourier transform: Definition - Properties - Fourier transform of some elementary functions - Convolution theorem - Fourier transform as a limit of Fourier Series - Applications to

### PDE.

INTEGRAL EQUATIONS: Volterra Integral Equations: Basic concepts – Relationship between Linear differential equations and Volterra integral equations - Resolvent Kernel of Volterra Integral equation - Solution of Integral equations by Resolvent Kernel- The Method of successive approximations - Convolution type equations, solution of integral differential equations with the aid of Laplace transformation. Fredholm Integral equations: Fredholm equations of the second kind, Fundamentals - Iterated Kernels, Constructing the resolvent Kernel with the aid of iterated Kernels - Integral equations with degenerate Kernels -Characteristic numbers and eigen functions, solution of homogeneous integral equations with degenerate Kernel - non-homogeneous symmetric equations- Fredholm alternative.

CALCULUS OF VARIATIONS: Extrema of Functionals: The variation of a functional and its properties - Euler's equation - Field of extremals - suffcient conditions for the Extremum of a Functional, conditional Extremum, Moving boundary problems - Discontinuous problems - one sided variations - Ritz method.

## Text books:

1. I. M. Gelfand and S. V. Fomin, Calculus of Variations, Dover, 2000.

2. Ram P Kanwal, Linear Integral Equations, Academic Press, 1971.

## **References:**

1. I. N. Sneddon, The Use of Integral Transforms, Tata McGraw Hill, 1972. 2. Porter D. and Stirling S. G., Integral Equations, A Practical Treatment, Cambridge University Press, 1990.

3. Gakhov F. D., Boundary Value Problems, Addision Wesley, 1966.

4. Muskhelishvilli N. I., Singular Integral Equations, Noordho\_, 1963.

5. M. L. Krasnov, G. K. Makarenko and A. I. Kiselev, Problems and Exercises in Calculus

of Variations, Imported Publishers, 1985.

6. Ram P Kanwal, Linear Integral Equations, Academic Press, 1971.

7. A. M. Wazwaz, A First Course in Integral Equations, World Scienti\_c, 1997. 8. F. B. Hildebrand, Methods of Applied Mathematics, Prentice Hall, 1965. Introduction, Cambridge University Press, 1995.

Code:MAT5015: Operator Theory		т	Р	Credit
Prerequisites: Real analysis, topology and functional analysis	3	2	0	4

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Elective

Course Type	Theory
Course Objective	The objective of this course is to introduce fundamental topics in operator theory. This course envisages to study compact operators, spectral theory of Banach space operators and Hilbert space operators.

Course	Upon completion of this course students will be able to understand:
Outcome(s)	Various operators on Hilbert spaces: self-adjoint, normal, unitary,
	isometry, partial isometry, projections, positive operators.; numerical
	range and numerical radius; Hilbert-Schmidt operators; eigen spectrum,
	Approximate eigen spectrum and resolvent set; spectral radius formula;
	spectral mapping theorem; spectrum of various operators on Hilbert
	spaces; Finite rank operators, compact operators; Riesz-Schauder theory
	for compact operators; spectral theorem for compact self-adjoint and
	compact normal operators; singular value decomposition of compact
	operators.

Dual space consideration: Representation of duals of the spaces c<sub>00</sub> with p-norms, c<sub>0</sub> and c with supremun-norm, I-p, C[a, b] and L<sup>p</sup>. Reflexity; Weak and weak\* convergences. Operations on Banach and Hilbert spaces: Compact operators between normed linear spaces; Integral operators as compact operators; Adjoint of operators between Hilbert spaces; Self adjoint, normal, unitary operators; Numerical range and numerical radius; Hilbert-Schmidt operators. Spectral results for Banach and Hilbert space operators; Eigen spectrum, Approximate eigen spectrum and resolvent; Spectral radius formula, Spectral mapping theorems; Riesz-Schauder theory; Spectral results for normal, self- adjoint, unitary operators; Functions of self-adjoint operators. Spectral representation of operators: Spectral theorem and singular value representation for compact self-adjoint operators; Spectral theorem for self-adjoint operators.

### Text books:

1. Conway J. B., A course in Functional Analysis, Springer-Verlag, 1990.

2. Rudin W., Functional Analysis, Tata Mcgraw-Hill, 1974.

### **References:**

B.V. Limaye, Functional Analysis, 2nd Edition, New Age International, 2008.
Edouard Goursat, A Course in Mathematical Analysis, Nabu Press, 2013.
Kreyszig, Introduction to Di\_erential Geometry and Reimannian Geometry, University of Toronto press, 1969.
A.E. Taylor and D.C. Lay, Introduction to Functional Analysis, 2nd Edition, Wiley, New York, 1980

Code:MAT5016: Operations Research	L	Т	Ρ	Credit
Prerequisites:	3	2	0	4

Course Category	Elective
Course Type	Theory
Course Objective	Operations research helps in solving problems in different environments

	that needs decisions. This module aims to introduce students to use quantitative methods and techniques for effective decisions— making; model formulation and applications that are used in solving business decision problems
Course Outcome(s)	Identify and develop operational research models from the verbal description of the real system. Understand the mathematical tools that are needed to solve optimisation problems. Use mathematical software to solve the proposed models. Develop a report that describes the model and the solving technique, analyse the results and propose recommendations in language understandable to the decision-making processes in Management and Engineering

Introduction, uses and limitations. Preliminaries - Convex functions, modeling, formulation of linear programming problems. Graphical method, theory of simplex method -Simplex Algorithm - Charnes M-Method - Two phase method, Computational complexity of simplex Algorithm - Karmarker's Algorithm. Duality in linear programming, Dual simplex method, Sensitivity analysis, Bounded variable problem,

Transportation problem, Integrity property, MODI Method, Degeneracy -Unbalanced problems. Assignment problem - Hungarian method - Routing problems Dynamic programming problem -Bellmann's optimality principle - Cargo loading problem - Replacement problem - Multistage production planning and allocation problem. Game theory - Rectangular Games - Two persons zero sum games - Pure and mixed strategies - 2 X n and m X2 games - Relation between theory of games and linear

### programming.

Critical path analysis - Probability consideration in PERT. Distinction between PERT and CPM -Resources Analysis in network scheduling - Time cost optimization algorithm - Linear programming formulation - Introduction to optimization softwares. Non –linear programming problems.

### Text books:

1. Frederick S. Hillier, Gerald J. Lieberman, Bodhibrata Nag and Preetam Basu, Introduction to Operations Research, McGraw-Hill, 10th Edition, 2017.

#### **References:**

1. M. S. Bazaara, J. J. Jarvis and H.D. Sherali, Linear programming and Network flows, John Wiley, 2nd Edition, 2009.

2. M. S. Bazaara, H. D. Sherali and C. M. Shetty, Nonlinear programming Theory and Algorithms, John Wiley, 2nd Edition, 2006.

3. Taha H. A., Operations Research - An Introduction, Prentice Hall India, 7th Edition, 2006. 4. Hadley G., Linear Programming, Narosa Book Distributors, 2002.

Code:MAT5017: Optimization Techniques and Control	L	Т	Р	Credit
Theory Prerequisites:	3	2	0	4

Course Category	Elective
Course Type	Theory

Course Objective	To provide a strong foundation on optimization techniques and its application in classical and modern control theory.
Course Outcome(s)	Formulate optimization problems on standard form from specifications on dynamics, constraints and control objective. In addition, be able to explain how various control objectives affect the optimal performance. Appreciate issues of robustness, optimality, architecture and uncertainty in control problems

Functions taking values in extended reals, proper convex functions, Subgradients, Directional derivatives, Conjugate functions, Conjugate duality. Gradient descent method, gradient projection method, Newton's method, Conjugate gradient method. Dynamic programming, Bellman's principle of optimality, Allocation problem, Cargo loading problem, Stage coach problem. Optimal control problem, Classical approach to solve variational problems, Pontryagin's maximum principle, Dynamic programming and maximum principle.

## Text books:

1. D. Liberzon, Calculus of variation and Optimal Control Theory: A Concise Introduction, Princeton University Press, 2012.

### **References:**

1. M. Avriel, Nonlinear Programming: Analysis and Methods, Dover Publications, New York, 2012.

2. O. Guler, Foundation of Optimization, Springer, 2010.

Code:MAT5018: Probability Theory	L	Т	Ρ	Credit
Prerequisites: Measure Theory.	3	2	0	4

Course Category	Elective
Course Type	Theory
Course Objective	To enable students to have an overview and thorough understanding of the modern probability theory.

Course	Upon completion of the subject, students will be able to: Apply the							
Outcome(s)	concepts of probability, conditional probability and conditional,							
	expectations. Calculate probabilities, moments and other related							

quantities based on given distributions. Understand and apply the laws of
large numbers and central limit theorems, martingale limit theory,
Brownian motion model.

Probability measures and random variables, pi and lambda systems, expectation, moment generating function, characteristic function, laws of large numbers, limit theorems, conditional contribution and expectation, martingales, infinitely divisible laws and stable laws.

## Text books:

1. Durrett R., Probability: Theory and Examples, Cambridge University Press, 4th Edition, 2010.

## **References:**

 Billingsley P., Probability and Measure, 3rd Edition, Wiley India, 2008.
Kallenberg O., Foundations of Modern Probability, 2nd Edition, Springer-Verlag 2002, 2. Walch L. Knowing the Odds: An Introduction to Probability, All

Verlag, 2002. 3. Walsh J., Knowing the Odds: An Introduction to Probability, AMS, 2012.

Code:MAT5019: Queueing Theory	L	Т	Ρ	Credit
Prerequisites: Basic Probability	3	2	0	4

Course Category	Elective
Course Type	Theory

Course Objective	To develop the modeling and mathematical skills to analytically determine computer systems and analytically determine computer systems and communication network performance. Students should be able to read and understand the current performance analysis and queueing theory literature upon completion of the course. Understand strengths and weaknesses of Queueing Models
Course Outcome(s)	Construct models in discrete and continuous time based on Markov Chains, describe and explain the theory of Markov Chains, describe and motivate Little's formula and its applications, describe and analyze basic Markov queuing models and situations to which they may be applied apply Markov models for selected applications.

Probability and random variable, discrete and continuous, univariate and multivariate distributions, moments, law of large numbers and central limit theorem (without proof). Poisson process, birth and death process, infinite and finite queueing models M/M/1, M/M/C, M/G/1, M/M/1/N, M/E/1, E/M/1, M/G/1/N, GI/M/1, and more complex non-Markovian queueing models - GI/G/1 queues, Multiserver Queues: M/M/c, M/G/c, GI/M/c modles, Erlang's loss system, Queues with finite populations: M/M/1/N/K, M/G/1/N/K etc. models and Engset formula, Concept bulk queues: M[X]/M/1, M/M[Y]/1, M/M(a, b)/1, M[X]/G/1,

GI[X]/M/1, M/G(a, b)/1, GI/M(a, b)/1 etc. queueing models. Priority queueing models, Vacation queueing models, Network of queues, finite processor sharing models, central server model of multiprogramming, performance evaluation of systems using queueing models. Concepts of bottleneck and system saturation point. Introduction to discrete time queues and its applications.

### Text books:

1. Gross D. and Harris C. M., Fundamentals of Queueing Theory, Wiley, 2012.

### **References:**

- 1. Kleinrock L., Queueing Systems Volume 1 : Theory, Wiley, 2013.
- 2. Kleinrock L., Computer Applications, Volume 2, Queueing Systems, Wiley, 2013.

Code:MAT5020: Stochastic Models and	L	Т	Ρ	Credit
Applications Prerequisites: Basic Probability	3	2	0	4

Course Category	Elective
Course Type	Theory
Course Objective	Upon completion of this course, students will: • understand the need for system models that capture random behavior to assess the risk of undesirable outcomes. • be able to model a number of important industrial and service systems and analyze those models to improve system performance. • be able to construct algorithmic solution strategies to explore system models that have been developed.
Course Outcome(s)	Students would acquire a rigorous understanding of basic concepts in probability theory. They would learn some important concepts concerning multiple random variables such as Bayes rule for random variables, conditional expectation and its uses etc. They would also learn stochastic processes, including Markov Chains and Poisson Processes. The course would provide the background needed to study topics such as Machine Learning, Adaptive Signal Processing, Estimation Theory etc

Probability spaces, conditional probability, independence, random variables, distribution functions, multiple random variables and joint distributions, functions of random variables, moments, characteristic functions and moment generating functions, conditional expectation, sequence of random variables and convergence concepts, laws of large numbers, central limit theorem, stochastic processes, Markov chains, Poisson process.

### Text books:

1. Ross S. M, Introduction to Probability Models, 10th Edition, Academic Press, 2012.

#### **References:**

 P. G. Hoel, S. C. Port and C. J. Stone, Introduction to Probability Theory, 1971.
P. G. Hoel, S. C. Port and C. J. Stone, Introduction to Stochastic Processes, 1972.

Code:MAT5021: Topological Dynamics	L	т	Р	Credit	
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## Prerequisites: Topology

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Course Category	Elective
Course Type	Theory
Course Objective	This course will lay the foundation to the discrete dynamical systems.
Course Outcome(s)	Students would acquire adequate knowledge in the theory of the topological dynamics, especially; Phase Portraits, Periodic Points and Stable Sets, Sarkovskii's Theorem, Hyperbolic, Attracting and Repelling Periodic Points, Families of Dynamical Systems, Bifurcation, Topological Conjugacy, The Logistic Function, Cantor Sets and Chaos, Period - Doubling Cascade, Symbolic Dynamics, Newton's Method, Numerical Solutions of Differential Equations, Complex Dynamics, Quadratic Family, Julia Sets, Mandelbrot Set, Topological Entropy, Attractors and Fractals, Theory of Chaotic Dynamical systems.

## Syllabus:

Phase Portraits, Periodic Points and Stable Sets, Sarkovskii's Theorem, Hyperbolic, Atracting and Repelling Periodic Points. Families of Dynamical Systems, Bifurcation, Topological Conjugacy.

The Logistic Function, Cantor Sets and Chaos, Period - Doubling Cascade. Symbolic Dynamics. Newton's Method. Numerical Solutions of Differential Equations. Complex Dynamics, Quadratic Family, Julia Sets, Mandelbrot Set. Topological Entropy, Attractors and Fractals, Theory of Chaotic Dynamical systems.

### Text books:

1. Richard A. Holmgren, A First Course in Discrete Dynamical Systems, Springer Verlag, 2000. 2. R. L. Devaney, Introduction to Chaotic Dynamical Systems, Westview press, 2003.

### **References:**

1 Michael Brin and Garrett Stuck, Introduction to Dynamical Systems, Cambridge University Press, 2002.

Code:MAT5022: Topological Groups	L	Т	Ρ	Credit
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Prerequisites: Topology and Measure Theory	3	2	0	4	
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Course Category	Elective
Course Type	Theory
Course Objective	This course will lay the foundation to Locally compact second countable spaces, Measure Theory on LCSC spaces and some basic knowledge in Topological groups.
Course Outcome(s)	Students would acquire adequate knowledge in Locally compact second countable spaces, Measure Theory on LCSC spaces, Measure Theory and Functional Analysis, Linear groups - some basic facts, Topological groups - basics, Characters, Dual groups, Sample results about the structure of LCSC abelian groups, Some Major Theorems (without proof) and their consequences, Abstract Fourier Transform. Peter-Weyn Theorem, Pontryagin Duality.

Locally compact second countable spaces, Measure Theory on LCSC spaces, Measure Theory and Functional Analysis, Linear groups - some basic facts, Topological groups - basics, Characters, Dual groups, Sample results about the structure of LCSC abelian groups, Some Major Theorems (without proof) and their consequences, Abstract Fourier Transform. Peter Weyn Theorem, Pontryagin Duality..

# Text books:

1. Sidney A. Morris, Pontryagin duality and the structure of locally compact abelian groups, Cambridge University Press, 1977.

2. P. J. Higgins, An Introduction to Topological Groups, London Mathematical Society, Cambridge University Press, 1975.

3. Nelson G. Markley, Topological Groups: An Introduction, Wiley, 2010.

### **References:**

1 H. Helson, Harmonic Analysis, Addison-wesley Publishers, 1983.

2. W. Rudin, Fourier Analysis on Groups, Wiely-Interscience, 1990.

Code:MAT5023: Introduction to Distribution Theory	L	т	Р	Credit	
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Prerequisites: 3 2 0 4
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Course Category	Elective
Course Type	Theory
Course Objective	The aim of the course is to introduce distribution theory, and its importace in solving for the theory of partial differential equations.

Course Outcome(s)	The students get familiarize with foundations of distribution theory: test functions, the concept of a distribution, distributions with compact support, operations on distributions, convolution, homogeneous distributions and the Fourier transform. Application of distribution theory with examples
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Test Function and Distributions: Introduction, Test Functions, Convergence in test function, Distribution, Operations on Distributions, Multiplication and Division of Distributions, Local properties of Distributions, A Boundedness property.

Convergence of Distributions: Introduction, Convergence of a sequence of Distributions, Convergence of a series of Distributions. Differentiation of Distributions, Introduction, Distributional Derivative, Derivative of the product, Derivative of a locally Integrable f unction. Convolution of Distributions: Introduction, Distribution of Compact Support, Direct Product of Distributions, Some Properties of the Direct product, Convolution, Properties of Convolution, Regularization of Distributions, Fundamental Solutions of Linear Differential Operators. Tempered Distribution and Fourier transforms: Introduction, The Space of Rapidly Decreasing Functions, The Space of Tempered Distributions, Multipliers in S'(R<sup>n</sup>), The Fourier Transform on L<sup>1</sup>(Rn),The Fourier Transform on S(R<sup>n</sup>), The Fourier Transform on SO(Rn), Properties of the Fourier Transform on S'(Rn), Convolution Theorem in S'(Rn), The Fourier Transform on E'(Rn), Applications

Sobolev Spaces: Introduction, Hilbert Space, The Sobolev Sapace H  $^{m,p}$ (Omega), The Sobolev Space H<sup>s</sup>(R<sup>n</sup>) Product and Convolution in H  $^{s}(R^{n})$ , The Space H<sup>-s</sup>(R<sup>n</sup>), The Sobolev Space H<sup>1</sup>, Sobolev Space of Order s. Extension theorem, Imbedding and completeness theorem, trace theory. Fundamental solution and Application to Elliptic Problems: Weak solution of elliptic boundary value problem (BVP), regularity of weak solutions, maximum principle, eigenvalue problems.

### Text books:

1. F.G. Friedlander, Introduction to the theory of distributions, Cambridge University Press, Cambridge, (1998).

2. Robert A. Adams, John J. F. Fournier, Sobolev spaces, Elsevier, 2003.

3. J.J. Duistermaat, Johan A.C. Kolk, Distributions: Theory and Applications, Springer Science & Business Media (2010).

4. Ram P. Kanwal, Generalized Functions: Theory and Applications, Springer Science & Business Media, (2004)

5. Svetlin G. Georgiev, Theory of Distributions, Springer (2010)

#### **References:**

1 L.C. Evans, Partial Di\_erential Equations, AMS, (2010)

2. W. Rudin, Functional Analysis, Mc Graw Hill, New York, (1973).

3. E. DiBenedetto, Real Analysis, Birkhauser, Boston, (2002)

4. S. Kesavan, Topics in Functional Analysis and Applications

5. S. Salsa, Partial Di\_erential Equations in Action. From Modelling to Theory, 2<sup>nd</sup>

Edition, Springer- Verlag Italia, (2015).

6. A.H.Zemanian, Distribution Theory and Transform Analysis

Code:MAT5051: Probability Theory	L	Т	Ρ	Credit
Prerequisites:	3	2	0	4

Course Category	Elective
Course Type	Theory
Course Objective	This course will lay the foundation to probability theory and statistical modelling of outcomes of real life random experiments through various statistical distributions.
Course Outcome(s)	To know different ways to describe the distribution of a random variable; to know methods for treating and describing limits of sequences of random variables; to be familiar with how filtrations and conditional expectations are used to represent information and can work with discrete time martingales; know the construction of Brownian motions and some of their most important properties.

Sample spaces, events, Probability axioms, Conditional Probability, Independent events, Baye's formula, Random Variables, Distribution functions, Marginal distributions, Conditional distribution, Stochastic Independence. Expectation, Conditional expectation, Conditional Variance. Moment generating functions, Cumulative generating functions.

Probability distributions: Binomial, Poisson, geometric, Uniform, exponential, Normal, gamma, beta - generating function, Mean, variance. Correlation, Regression, Multiple and Partial Correlations. Probability density function - Properties - t distributions, f distributions, Chi square distribution.

Test for means, Variances and attributes - large sample tests. Analysis of Variance: One way and two way classifications, Complete Randomized blocks, Randomized Block Design, Latin Square Design.

Estimation: Point estimation, Characteristics of estimation, Interval estimation. Interval estimates of Mean, Standard deviation, proportion, difference in Means and ratios of Standard deviations. Time series analysis: Trend and Seasonal variations, Box Components of time Series, measurement of trend - linear and Second degree Parabola.

Statistical quality control, Statistical basis for control charts, Control limits, Control Charts for variables. X Charts, R Charts, Charts for defective - P, nP Charts, charts for defects, C Charts.

### Text books:

1. K. S. Trivedi, Probability and Statistics with reliability, queueing and Computer applications, Wiely, 2016.

#### **References:**

1 Montogomery D C, and Johnson. A, Forecasting and time Series analysis, McGraw- Hill, 1976. 2. Sren Bisgaard and Murat Kulahci, Time Series Analysis and Forecasting by Example, John Wiley & Sons, 2011.

3. Dale H. Bester\_eld, Quality Control, Pearson Education India, 2004.

4. Box G. E. P. and Jenkins G. M., Time series analysis, Holden-Day, 1976.

Code:MAT5052: Operations Research	L	Т	Ρ	Credit
Prerequisites:	3	2	0	4

Course Category	Elective
Course Type	Theory
Course Objective	To introduce students to use quantitative methods and techniques for effective decisions—making; model formulation and applications that are used in solving business decision problems
Course Outcome(s)	Solve Linear Programming Problems ; Solve Transportation and Assignment Problems ; Understand the usage of game theory and Simulation for Solving Business Problems

Linear Programming - Formulation of Linear programming problems - Various definitions -Statements of basic theorems and different properties. Graphical Method for twodimensional problems, Phase I and Phase II of the Simplex Method - Duality and Shadow Price - Sensitivity analysis - transportation Problems - Assignment Problem.

Queueing theory: Characteristics of queueing Systems - Steady State M/M/1, M/M/C and M/M/K queueing Models.

Game theory - Rectangular Games - Two persons zero sum games - Pure and mixed strategies - 2 X n and m X 2 games - Relation between theory of games and linear programming

Critical path analysis - Probability consideration in PERT. Distinction between PERT and CPM -Resources Analysis in networking scheduling - Time cost optimization algorithm - Linear programming formulation - Introduction to optimization softwares. Non-linear programming problems.

### Text books:

1. Frederick S. Hillier, Gerald J. Lieberman, Bodhibrata Nag and Preetam Basu, Introduction to Operations Research, McGraw-Hill Education, 10th Edition, 2017.

### **References:**

- 1. M.S. Bazaara, J.J. Jarvis and H.D. Sherali, Linear programming and Network ows , John Wiley, 2nd Edition, 2009.
- 2. M.S. Bazaara, H.D. Sherali and C.M. Shetty, Nonlinear programming Theory and Algorithms, John Wiley, 2nd Edition, 2006.

3. Taha, H. A., Operation Research- An Introduction , Prentice Hall India, 7th Edn.,

2006 4. Hadley, G., Linear Programming, Narosa Book Distributors, 2002.

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January 25, 2021 A. K. Nandakumaran